Kuwait University
Department of Mathematics & Computer Science

Math 102 — December 15, 2007
Calculus II — Second Midterm — Time: 75 minutes

A. Evaluate the following integrals: [3 pts each]

1. $\displaystyle\int \csc x \cot x \ln(\sin x)\, dx$

2. $\displaystyle\int \tan^{3/2} x \sec^4 x\, dx$

3. $\displaystyle\int \cos 3x \sin^2 x\, dx$

4. $\displaystyle\int \frac{\cosh x}{1+\sin(\sinh x)}\, dx$

5. $\displaystyle\int \frac{(x+2)^2}{\sqrt{-x^2-4x}}\, dx$

6. $\displaystyle\int \frac{\sqrt[4]{x}}{x\left(\sqrt[3]{x}+\sqrt{x}\right)}\, dx$

B. Determine whether the following integrals converge or diverge; if convergent, find the value.

7. $\displaystyle\int_{-\infty}^{\infty} e^{-x}\, dx$ [3 pts]

8. $\displaystyle\int_{1}^{2} \frac{1}{x^2\sqrt{x-1}}$ [4 pts]

1. Integration by parts with $u = \ln(\sin x)$ and $dv = \csc x \cot x\, dx$ yields

$$\int \csc x \cot x \ln(\sin x)\, dx = -\csc x \ln(\sin x) + \int \csc x \cot x\, dx$$

which reduces to $\csc x \ln(\csc x) - \csc x + C$.

2. Piece of cake: put $u = \tan x$

$$\int \tan^{3/2} x \sec^4 x\, dx = \int \tan^{3/2} x\, (1 + \tan^2 x) \sec^2 x\, dx = \int (u^{3/2} + u^{7/2})\, du$$

3. Use the identity $2 \cos a \cos b = \cos(a+b) + \cos(a-b)$ and the half angle formula for $\sin^2$:

$$\cos 3x \sin^2 x = \frac{1}{2} \cos 3x\, (1 - \cos 2x) = \frac{1}{2} \cos 3x - \frac{1}{4} (\cos 5x + \cos x)$$

Then

$$\int \ldots = \frac{1}{6} \sin 3x - \frac{1}{20} \sin 5x - \frac{1}{4} \sin x + C$$

4. Put $z = \sinh x$ and you'll have

$$\int \frac{\cosh x}{1 + \sin(\sinh x)}\, dx = \int \frac{dz}{1 + \sin z}$$

Weierstrass substitution reduces this to

$$\int \frac{2}{(1+u)^2}\, du = -\frac{2}{1+u} + C, \qquad u = \tan \frac{z}{2} = \tan \frac{\sinh x}{2}$$

5. Note that $-x^2 - 4x = 4 - (x+2)^2$. Let $x + 2 = 2 \sin \theta$. The integral becomes

$$\int \frac{(x+2)^2}{\sqrt{4 - (x+2)^2}}\, dx = \int 4 \sin^2 \theta\, d\theta = \int 2\, (1 - \cos 2\theta)\, d\theta \quad \text{etc.}$$

6. Put $x = u^{12}$ to get

$$\int \frac{\sqrt[4]{x}}{x\, (\sqrt[3]{x} + \sqrt{x})}\, dx = \int \frac{12}{u^2\, (3 + u^2)}\, du = 4 \int \left[\frac{1}{u^2} - \frac{1}{3 + u^2}\right] du$$

which becomes

$$-4 \left[\frac{1}{u} + \frac{1}{\sqrt{3}} \tan^{-1} \frac{u}{\sqrt{3}}\right] + C \ \text{ etc.}$$

7. Divergent!

8. Put $x = \sec^2 \theta$. Then

$$\int \frac{1}{x^2 \sqrt{x-1}}\, dx = \int 2 \cos^2 \theta\, d\theta = \theta + \sin \theta \cos \theta$$

which, in terms of x, becomes

$$\lim_{s \to 1^+} \sec^{-1} \sqrt{x} + \frac{\sqrt{x-1}}{x} \Bigg|_s^2 = \frac{\pi}{4} + \frac{1}{2}$$